(3) A point x has equal distance to a and b if and only if

$$(x-a)\cdot(x-a) = (x-b)(x-b)$$

expanding

$$x^2 - 2ax + a^2 = x^2 - 2xb + b^2.$$

Rearranging

$$a^2 - b^2 = -2xb + 2ax$$

$$b^2 - a^2 = 2bx - 2ax$$

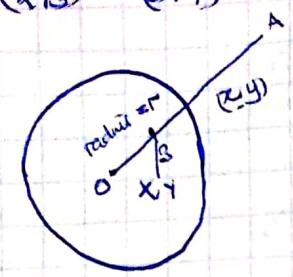
$$(b\cdot b - a\cdot a) = 2x(b-a)$$

$$(b-a)x = \frac{1}{2}(b\cdot b - a\cdot a).$$

$(b-a)x = \frac{1}{2}(b\cdot b - a\cdot a)$ which is the equation of a line.

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2. (2,3) (5,7)



Point A is mapped to point B where $OA \times OB = r^2$
often $r = 1$.
Using $r = 1$ in this question.

$$r^2 = \sqrt{x^2+y^2} \cdot \sqrt{x^2+y^2}$$

$$r^2 = \sqrt{2^2+3^2} \cdot \sqrt{5^2+7^2}$$

$$r^2 = \sqrt{962}$$

$$r = 5.57 \text{ units}$$

Centre co-ordinates $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{xk^2}{x^2+y^2} \\ \frac{yk^2}{x^2+y^2} \end{bmatrix}$

$$\frac{xk^2}{x^2+y^2} = \frac{5(\sqrt{962})}{74} = 2.096$$

$$\frac{yk^2}{x^2+y^2} = \frac{7 \times \sqrt{962}}{74} = 2.934.$$

$(X, Y) = (2.096, 2.934)$ – centre coordinates

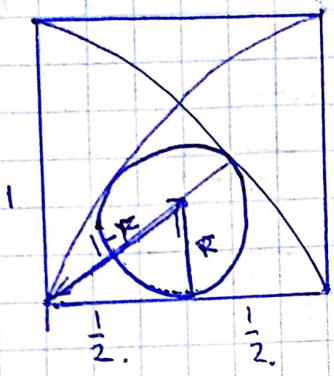
④ Sq. length = 1
quarter radius = 1
Half circle radius = $\frac{1}{2}$.
Radii of disks

Radius of the smaller disk (orange colour)

$$(R-d)^2 = R^2 - (R/2)^2.$$
$$(R-d) = \sqrt{(3/4)} * R.$$
$$d = R(1 - \sqrt{3}/2$$
$$r = \frac{d}{2} = \frac{R}{2}(1 - \sqrt{3}/2). \quad R = 1$$
$$\therefore r = \frac{1}{2}(1 - \sqrt{3}/2)$$

$$(R-d)^2 = R^2 - (R/2)^2. \quad \text{but } R = 1$$
$$(1-d)^2 = 1 - (\tfrac{1}{2})^2$$
$$(1-d)^2 = 3/4$$
$$1-d = \sqrt{3}/4$$
$$d = 1 - \sqrt{3}/4$$
$$r = \frac{1}{2}(1 - \sqrt{3}/4) = 0.06698 \text{ units}$$

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From the described triangle.

Pythagoras theorem.

$(1-R)^2 = R^2 + \frac{1}{2}^2$

$1 + R^2 - 2R. = R^2 + 1/4.$

Rearranging

$1 + R^2 - 2R = R^2 + 1/4.$

$R^2 - R^2 - 2R = 1/4 - 1$

$+2R = +3/4$

$R = 0.375$ units

Radius of the larger disk $= 0.375$ units

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